Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel Level 3 GCE**

| Centre Number | Candidate Number |
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# Thursday 14 May 2020

Afternoon | Paper Reference **8FM0/26**

# Further Mathematics

**Advanced Subsidiary**
**Further Mathematics options**
**26: Further Mechanics 2**
**(Part of option J)**

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of $g$ is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 3 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

**1.**

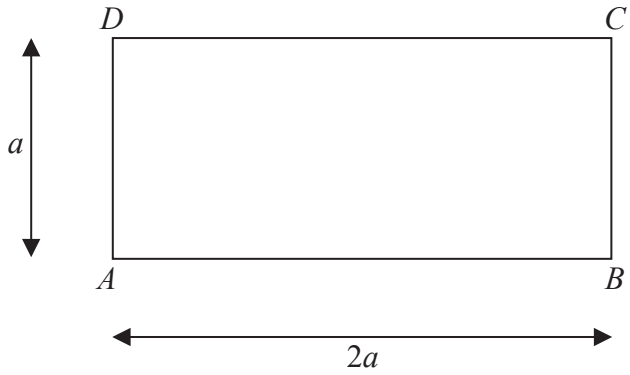



**Figure 1**

Figure 1 shows a uniform rectangular lamina $ABCD$ with $AB = 2a$ and $AD = a$ The mass of the lamina is $6m$.

A particle of mass $2m$ is attached to the lamina at $A$, a particle of mass $m$ is attached to the lamina at $B$ and a particle of mass $3m$ is attached to the lamina at $D$, to form a loaded lamina $L$ of total mass $12m$.

(a) Write down the distance of the centre of mass of $L$ from $AB$. You must give a reason for your answer.

**(2)**

(b) Show that the distance of the centre of mass of $L$ from $AD$ is $\frac{2a}{3}$

**(3)**

A particle of mass $km$ is now also attached to $L$ at $D$ to form a new loaded lamina $N$.

(c) Show that the distance of the centre of mass of $N$ from $AB$ is $\frac{(k+6)a}{(k+12)}$

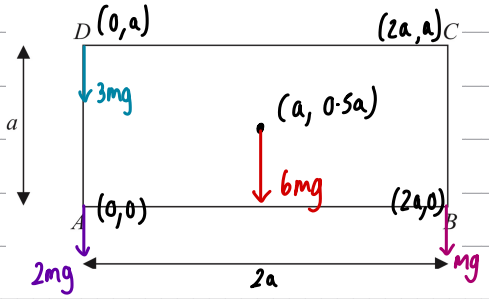
**(4)**

When $N$ is freely suspended from $A$ and is hanging in equilibrium, the side $AB$ makes an angle $\alpha$ with the vertical, where $\tan\alpha = \frac{3}{2}$

(d) Find the value of $k$.

**(6)**

**Question 1 continued**




moments = force x perpendicular distance
The sum of moments is equal to the overall moment acting through the COM.
Mathematically → $\sum m_i x_i = \bar{x} \sum m_i$
Where m = force
x = perpendicular distance

Using A as (0,0)

$$6mg\begin{pmatrix} a \\ 0{\cdot}5a \end{pmatrix} + 2mg\begin{pmatrix} 0 \\ 0 \end{pmatrix} + mg\begin{pmatrix} 2a \\ 0 \end{pmatrix} + 3mg\begin{pmatrix} 0 \\ a \end{pmatrix} = 12mg\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\begin{pmatrix} 8m \\ 6m \end{pmatrix} = \begin{pmatrix} 12m\bar{x} \\ 12m\bar{y} \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} \frac{2}{3}a \\ \frac{1}{2}a \end{pmatrix}$$

b) ∴ distance from AD = $\frac{2a}{3}$

a) ∴ distance from AB = $\frac{a}{2}$

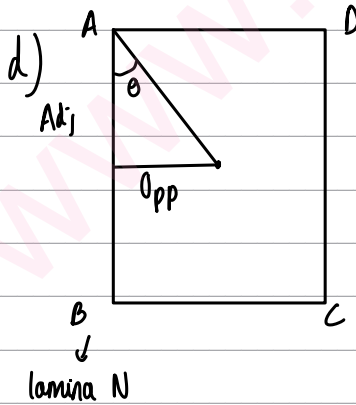
c) Using lamina L and new load placed at D → (0,a)

$$12mg\begin{pmatrix} \frac{2}{3}a \\ \frac{1}{2}a \end{pmatrix} + kmg\begin{pmatrix} 0 \\ a \end{pmatrix} = (12+k)mg\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$8a = (12+k)\bar{x}$ $\qquad$ $6a + ka = (12+k)\bar{y}$

$\bar{x} = \frac{8a}{12+k}$ $\qquad$ $\bar{y} = \frac{a(6+k)}{12+k}$

d)




lamina N

$\tan\alpha = \frac{3}{2} = \frac{\bar{x}}{\bar{y}}$

$\bar{x}$ = adj

$\bar{y}$ = opp

$\frac{3}{2} = \frac{a(6+k)}{12+k} \div \frac{8a}{12+k}$

$\frac{3}{2} = \frac{6+k}{8}$

$12 = 6+k$

$k = 6$

**Question 1 continued**

**Question 1 continued**

**(Total for Question 1 is 15 marks)**

**2.**

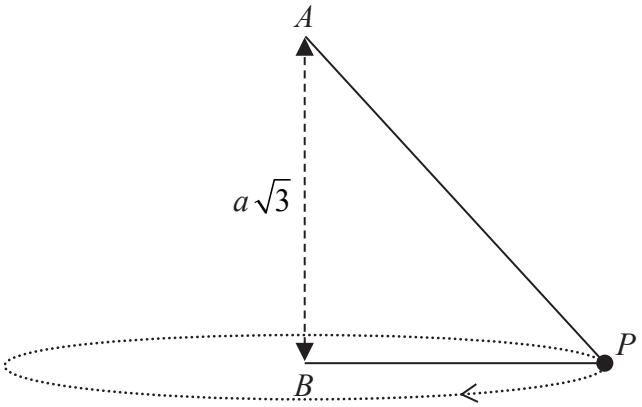



**Figure 2**

One end of a string of length $3a$ is attached to a point $A$ and the other end is attached to a point $B$ on a smooth horizontal table. The point $B$ is vertically below $A$ with $AB = a\sqrt{3}$ A small smooth bead, $P$, of mass $m$ is threaded on to the string. The bead $P$ moves on the table in a horizontal circle, with centre $B$, with constant speed $U$. Both portions, $AP$ and $BP$, of the string are taut, as shown in Figure 2.

The string is modelled as being light and inextensible and the bead is modelled as a particle.

(a) Show that $AP = 2a$

**(2)**

(b) Find, in terms of $m$, $U$ and $a$, the tension in the string.

**(4)**

(c) Show that $U^2 < ag\sqrt{3}$

**(5)**

(d) Describe what would happen if $U^2 > ag\sqrt{3}$

**(1)**

(e) State briefly how the tension in the string would be affected if the string were not modelled as being light.

**(1)**

a) Let $AP = x$

$(a\sqrt{3})^2 + (3a - x)^2 = x^2$

$3a^2 + x^2 - 6ax + 9a^2 = x^2$

$12a^2 - 6ax = 0$

$6a(2a - x) = 0$

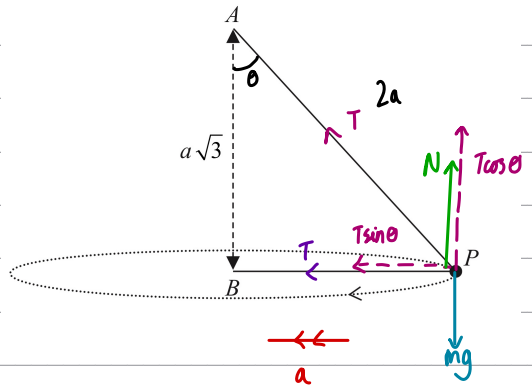
$2a - x = 0$

$x = 2a = AP$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\sin\theta = \frac{1}{2}$

**Question 2 continued**

b)




Since string is light $T = T$

Resolving forces horizontally

$T + T\sin\theta = \frac{mU^2}{a}$

$T + \frac{1}{2}T = \frac{mU^2}{a}$

$\frac{3}{2}T = \frac{mU^2}{a}$

$T = \frac{2mU^2}{3a}$

$a = \frac{v^2}{r}$ where $v$ = constant speed, $r$ = radius of motion

c) Resolving forces vertically

$T\cos\theta + N = mg$

$T\left(\frac{\sqrt{3}}{2}\right) + N = mg$ Substituting expression found in b)

$\frac{2mU^2}{3a}\left(\frac{\sqrt{3}}{2}\right) + N = mg$

$\frac{2\sqrt{3}mU^2}{6a} + N = mg$

$N = mg - \frac{2\sqrt{3}mU^2}{6a}$

On the table so $N > 0$

$0 < mg - \frac{2\sqrt{3}mU^2}{6a}$

$\frac{2\sqrt{3}U^2}{6a} < mg$

$\frac{\sqrt{3}U^2}{3a} < g$

$\sqrt{3}U^2 < 3ag$

$U^2 < \frac{3ag}{\sqrt{3}}$

$\therefore U^2 < ag\sqrt{3}$

d) if $U^2 > ag\sqrt{3}$, $N$ would be 0 (or less) which means the bead would no longer be on the table as there is no normal contact force

e) Tension on either side of the string (AP and BP) would not be the same as the mass would vary as the lengths AP and BP are different

**Question 2 continued**

**Question 2 continued**

**(Total for Question 2 is 13 marks)**

**3.** At time $t = 0$, a toy electric car is at rest at a fixed point $O$. The car then moves in a horizontal straight line so that at time $t$ seconds ($t > 0$) after leaving $O$, the velocity of the car is $v\,\text{m s}^{-1}$ and the acceleration of the car is modelled as $(p + qv)\,\text{m s}^{-2}$, where $p$ and $q$ are constants.

When $t = 0$, the acceleration of the car is $3\,\text{m s}^{-2}$

When $t = T$, the acceleration of the car is $\frac{1}{2}\,\text{m s}^{-2}$ and $v = 4$

(a) Show that

$$8\frac{dv}{dt} = (24 - 5v)$$

**(6)**

(b) Find the exact value of $T$, simplifying your answer.

**(6)**

a) $a = p + qv$

Boundary conditions $t=0, v=0, a=3$

Substituting into 'a'

$a = p + qv$

$3 = p$

$t=T, v=4, a=\frac{1}{2}$

Substituting into 'a'

$a = 3 + qv$ Using $p=3$

$\frac{1}{2} = 3 + 4q$

$4q = -\frac{5}{2}$

$q = -\frac{5}{8}$

$\therefore a = 3 - \frac{5}{8}v$

$a = \frac{dv}{dt} \Rightarrow 3 - \frac{5}{8}v = \frac{dv}{dt}$ ↓ multiplying by 8 on both sides

$\therefore 24 - 5v = 8\frac{dv}{dt}$

b) $\int 1\,dt = 8\int \frac{1}{24-5v}\,dv$

$t + C = -\frac{8}{5}\ln|24-5v|$

where $C$ is a constant

$t=0, v=0 \Rightarrow C = -\frac{8}{5}\ln 24$

$\therefore t = -\frac{8}{5}\ln|24-5v| + \frac{8}{5}\ln 24$

at $T$, $v=4$ so $T = -\frac{8}{5}\ln|24-5(4)| + \frac{8}{5}\ln 24 = -\frac{8}{5}\ln(4) + \frac{8}{5}\ln(24)$

$T = \frac{8}{5}\ln(6)$

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 12 marks)**

**TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS**